STRENGTH DISTRIBUTION OF ELEMENTARY FLAX FIBRES DUE TO MECHANICAL DEFECTS

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ABSTRACT

Flax fibres are finding non-traditional applications as reinforcement of both inorganic and polymer matrix composite materials. The tensile strength of fibres is an important factor in ensuring efficient reinforcement. The mechanical properties of fibres are affected by the natural variability in plant as well as the damage sustained during processing, and thus have considerable variability. This necessitates statistical treatment of fibre characteristics. The strength of elementary flax fibres, produced by different manufacturers, has been characterised at several fibre lengths by standard tensile tests. It has been demonstrated that the two-parameter Weibull distribution fails to capture the length dependence of natural fibre strength. Alternative strength distributions, based on the assumption that the presence of defects limits fibre strength, are considered and found to provide satisfactory agreement with test results.

KEYWORDS

Flax; fibre; strength; distribution.

INTRODUCTION

The efficiency of fibre reinforcement in a composite material is determined primarily by the mechanical properties of reinforcing fibres and their adhesion to matrix. While the scatter of fibre stiffness exerts limited effect on composite properties, fibre strength distribution affects both strength and toughness of a composite. Weakest-link character of fibre failure is reflected in the commonly used Weibull distribution of fibre strength (Weibull, 1939)

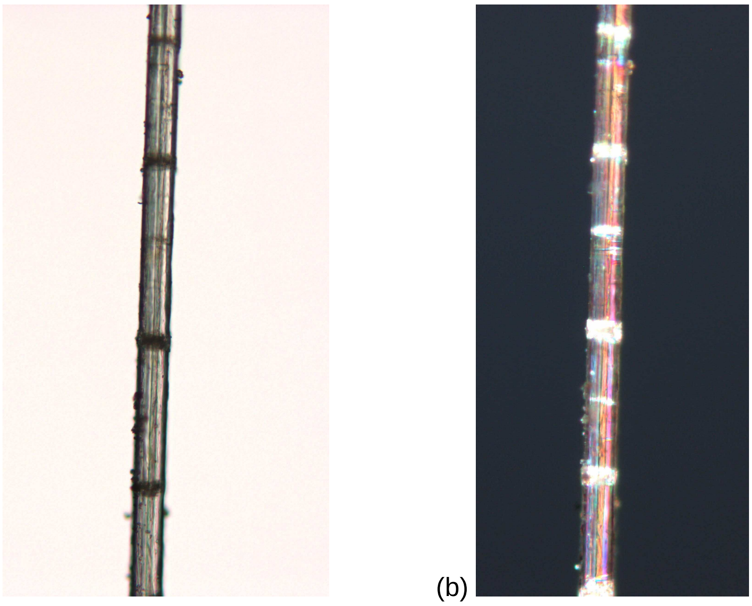
$$F(\sigma) = 1 - \exp\left[-\frac{l}{l_0}\left(\frac{\sigma}{\beta}\right)^{\alpha}\right] \quad (1)$$

where l stands for fibre length, l_0 is a normalizing parameter, □ is the tensile stress at fibre failure, and □, □ designate Weibull shape and scale parameters, respectively. However, it has been shown that the two-parameter Weibull distribution, Eq. (1), does not comply with the experimental data of flax fibre strength at different gauge lengths (Andersons et al., 2005, Zafeiropoulos and Baillie, 2007). Instead, a modified Weibull distribution

$$F(\sigma) = 1 - \exp\left[-\left(\frac{l}{l_0}\right)^{\gamma}\left(\frac{\sigma}{\beta}\right)^{\alpha}\right] \quad (2)$$

is found to agree with elementary flax fibre strength (Andersons et al., 2005). The physical origin of the distribution Eq. (2) is related to inter-fibre variation of strength characteristics. It has been demonstrated theoretically (Curtin, 2000) and experimentally (Andersons et al., 2002) that the distribution [Eq. (2)] for fibre batch is obtained if each of the fibres possesses Weibull strength distribution Eq. (1), but the parameters of Weibull distribution for individual fibres differ. Berger and Jeulin (2003) attribute Eq. (2) to the presence of a large-scale fluctuation of the density of defects (flaws) in fibres.

Elementary flax fibres may contain cell-wall defects, i.e. local misalignments of cellulose microfibrils, originating during growth and processing of flax. Such defects are variously called dislocations, kink bands, nodes, or slip planes (Nyholm et al., 2001). Dislocations are seen as bright zones in polarised light microscopy (e.g. Thygesen, 2007) crossing fibre and oriented roughly perpendicularly to its axis; the largest of them can also be discerned without polarisers as seen in Fig. 1. Fibre strength is reduced by the presence (Bos et al., 2002) and amount (Davies and Bruce, 1998) of dislocations. Fibre failure in tension initiates within a dislocation zone (Baley, 2004). Therefore it appears reasonable to derive and apply a strength distribution that accounts for the presence of dislocations in the fibre.



(a) (b)

Figure 13 – Dislocations in an elementary flax fibre as revealed by optical microscopy in transmitted non-polarised (a) and polarised (b) light in the same fibre fragment.

Notably, in one of the original derivations of the weakest link strength distribution presented in Weibull (1939), the number of defects per unit volume was treated as a continuous variable when obtaining an analogue of Eq. (1). As opposed to such a continuum treatment of the effect of defects, a weakest-link strength distribution has also been derived assuming that a discrete, finite number of defects is present in a body (Todinov, 2000). Distribution functions combining the features of weakest link and random defect models have been proposed and applied to describe fibre strength scatter (Knoff, 1993, Paramonov and Andersons, 2006). In this study, we derive a strength distribution function explicitly accounting for the presence of defects in fibres and evaluate its applicability to elementary flax fibre strength data at different gauge lengths.

EXPERIMENTAL

Two types of elementary flax fibres have been tested. The fibres produced by FinFlax Oy (Finland) are designated by A and Ekotex (Poland) by B in the following. The test procedure of ASTM D 3379-75 Standard was followed. Single fibres were manually separated from fibre bundles. Fibre ends were glued onto a paper frame. Three gauge length specimens were prepared with free fibre length of 5, 10 or 20 mm respectively. Tension tests were carried out on an electromechanical tensile machine equipped with mechanical grips. During mounting the specimens were handled only by the paper frame. Upon clamping of the ends of the paper frame by grips of the test machine, both sides of the frame were carefully cut in the middle. The tests were displacement-controlled with the loading rate of 0.5 mm/min. Fibre A test results have been reported in Andersons et al. (2005).

Fibre diameter was evaluated from observations under optical microscope or micrographs as the average of five apparent diameter measurements taken at different locations along the fibre. Olympus BX51 microscope was used. Micrographs of fibre B revealing the presence of dislocations are shown in Fig. 1.

Defect-governed fibre strength distribution

Limited number of defects

Consider a fibre containing a random number, *k*, of defects with distribution mass function p_k. The distribution of defect strength (i.e. the stress at which fibre breaks at a given defect) is designated by $F_d(\sigma)$, while the strength distribution of defect-free fibre is $F_{nd}(\sigma)$. Then the survival probability of the fibre is given by the product of corresponding probabilities for defect-free fibre, $1-F_{nd}(\sigma)$, and for defects. The latter is expressed as a sum $\sum_{k=0}^{n} p_k (1-F_d(\sigma))^k$ (see e.g. Paramonov and Andersons, 2007, 2008), where *n* is the maximum number of defects. Finally, strength distribution of the fibre reads as

$$F(\sigma) = 1-(1-F_{nd}(\sigma))\sum_{k=0}^{n} p_k (1-F_d(\sigma))^k \ . \tag{3}$$

It appears natural to assume binomial distribution for the number of defects

$$p_k = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \ . \tag{4}$$

Such a choice can be interpreted as follows: there are *n* spots (or links) present along the fibre that can contain a defect with probability *p*, and be defect-free with probability 1-*p*.
Inserting Eq. (4) in Eq.(3) and performing summation, we obtain

$$F(\sigma) = 1-(1-F_{nd}(\sigma))(1-pF_d(\sigma))^n \ . \tag{5}$$

It is likely that, at a given applied stress, the failure at a location of a defect is much more probable than failure of an intact part of the fibre. Then Eq. (5) can be simplified as follows

$$F(\sigma) = 1-(1-pF_d(\sigma))^n \ . \tag{6}$$

Taking the two-parameter Weibull distribution for the random strength of defects

$$F_d(\sigma) = 1-\exp\left[-\left(\frac{\sigma}{b}\right)^a\right] \tag{7}$$

we finally obtain

$$F(\sigma) = 1-\left(1-p\left(1-\exp\left[-\left(\frac{\sigma}{b}\right)^a\right]\right)\right)^n \ . \tag{8}$$

The derived fibre strength distribution, Eq. (8), comprises four parameters. The parameters *a*, *b* characterising defect strength distribution Eq. (7) are to be determined using mechanical tests of the fibres whereas the parameters *p*, *n* of the defect number distribution Eq. (4) could, in principle, be evaluated by non-destructive means.
Note that the length of fibre enters strength distribution via parameter *n*, i.e. the maximum number of defects in a fibre. In the following, we consider two specific relations of *n* and fibre length.

Large number of defects

The average number of defects, λ, according to the binomial distribution, Eq. (4), is given by

$$\lambda = pn \ . \tag{9}$$

Consider a fibre with unlimited maximum number of defects n but fixed λ. The corresponding fibre strength distribution can be obtained from Eq. (6) by taking the limit $n \to \infty$. To this end, the second term in the right-hand side of Eq. (6) is expressed as follows, taking into account Eq. (9)

$$(1 - pF_d(\sigma))^n = \left[\left(1 - \frac{\lambda F_d(\sigma)}{n}\right)^{\frac{n}{\lambda F_d(\sigma)}}\right]^{\lambda F_d(\sigma)}. \quad (10)$$

The limit of the expression in square brackets in the rhs of Eq. (10) is easily determined by applying the relation $\lim_{x\to\infty}(1 + c/x)^x = e^c$. Finally, the limit of fibre strength distribution function Eq. (6) at $n \to \infty$ is as follows:

$$F(\sigma) = 1 - \exp[-\lambda F_d(\sigma)]. \quad (11)$$

It is seen that Eq. (11) closely resembles Weibull distribution. It can be transformed into Eq. (1) by assuming the average number of defects proportional to the fibre length (i.e. constant linear density of defects □), $\lambda = l\rho$, and expanding $F_d(\sigma)$, $F_d(\sigma) \sim (\sigma/b)^a$.

Models of defect densit

Defect density scatter between fibres

Consider the maximum number of defects proportional to fibre length, $n = [l\rho]$, for each fibre, with □ varying randomly between fibres. (Here and below $[x]$ designates the integer part of *x*.) Then the strength distribution of fibre batch, each fibre of which is characterised by strength distribution Eq. (6), can be expressed as

$$F(\sigma) = 1 - \int_0^\infty (1 - pF_d(\sigma))^{n(\rho)} f(\rho) d\rho \quad (12)$$

where $f(\rho)$ designates the distribution density of the random variable □.
Eq. (12) should apply to fibres with length-independent density of defects that is constant along each fibre but differs between fibres.

Fibre length dependent defect density

Assume that the maximum number of defects in a fibre is a power function of its length

$$n = \lfloor n_0 (l/l_0)^\gamma \rfloor. \quad (13)$$

Then fibre strength distribution function is obtained as follows, by combining Eqs. (6) and (13), and assuming defect strength distribution Eq. (7)

$$F(\sigma) = 1 - \left(1 - p\left(1 - \exp\left(-\left(\frac{\sigma}{b}\right)^a\right)\right)\right)^{n(l)}. \quad (14)$$

Eq. (14) would apply if, e.g., there is an inherent mechanism of censoring that facilitates specific selection of fibres for tests at different gauge length. For example, it is well known that elementary flax fibres (being individual bast cells of flax) vary in length in a wide range. It is plausible that the geometrical dimensions of a cell correlate with defect density due to different stresses experienced by cells during growth and also during processing. Since fibres for short gauge length tests can be obtained from cells of different lengths while fibres for long gauge length tests can only come from relatively long cells, the difference in cell characteristics would also be reflected, to some extent, in fibre defect density and gauge length relation.

RESULTS AND DISCUSSION

The strength distributions of elementary flax fibres of types A and B are shown in Fig. 2; the mean strength and the standard deviation (STD) of strength as a function of fibre type and length are presented in Table 1. It is seen that fibres A are somewhat stronger than B for all the gauge lengths considered while the variation of strength, characterised by the ratio of STD to mean strength, is roughly comparable for both fibre types.

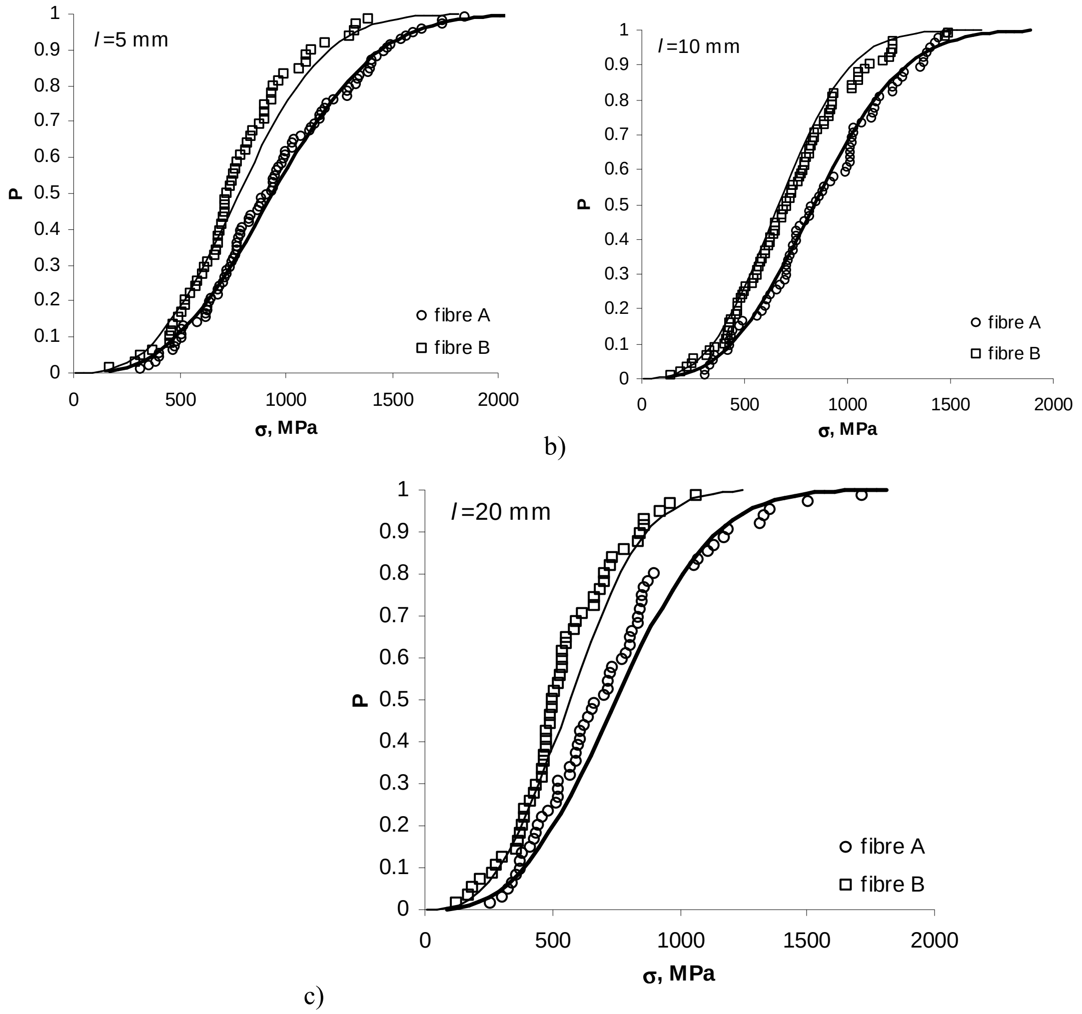


Figure 14 – Strength distribution of elementary flax fibres at 5 mm (a), 10 mm (b), and 20 mm (c) gauge length. Approximation by Eq. (14) is plotted by thick (fibres A) and thin (fibres B) solid lines.

Table 6 –Mean strength and standard deviation of strength as a function of fibre length.

Fibre type	A			B		
Fibre length, mm	5	10	20	5	10	20
Mean strength, MPa	960	870	737	763	722	540
Strength STD, MPa	359	320	320	266	287	207

A systematic study of defect distribution in the fibres would provide information necessary for the selection of an appropriate defect model and fibre strength distribution function. As such information is not yet available, we proceed by approximating the test results by theoretical distribution Eq. (14) in order to evaluate its applicability.

Choosing the parameter $l_0 = 1$ mm, n_0 in Eq. (13) becomes the maximum number of defects per 1 mm length of fibre. It is roughly estimated at n_0=10. Probability p = 0.5 is somewhat arbitrarily assumed. The remaining three parameters of the strength distribution Eq. (14) are evaluated by the maximum likelihood method using test data at all three gauge lengths. The parameter values thus obtained are shown in Table 2, and the theoretical distribution functions plotted in Fig. 2.

Table 7 – Parameters of fibre strength distribution Eq. (14).

Fibre/Parameter	a	b, MPa	γ	n_0	p
A	2.87	2400	0.46	10	0.5
B	2.83	2240	0.64	10	0.5

Note that, in view of the relatively large n values, the relation $n = n_0 \left(l/l_0 \right)^{\gamma}$ was used instead of the integer part of it stipulated by Eq. (13). The strength distribution of fibres A has been approximated by the modified Weibull distribution, Eq. (2), in Andersons et al. (2005). The γ parameter value of Eq. (2) reported in Andersons et al. (2005) coincides with that of Eq. (14), Table 1, which suggests that n values are high enough for the asymptotic relation of Eq. (11) to closely agree with Eq. (14).

It is seen in Fig. 2 that the distribution function Eq. (14) is capable of capturing not only the scatter of fibre strength at a fixed gauge length but also the dependence of fibre strength on gauge length. It is interesting to note that the parameters of defect strength distribution a and b are very close for fibres A and B, see Table 2. The lower strength of fibres B is therefore mainly due to the larger number of defects present, as follows from Eq. (13) and the higher value of γ parameter for fibres B. Experimental confirmation of the relation between the distribution of dislocations in flax fibres and fibre strength distribution is the subject of further research.

CONCLUSION

Experimental observations of flax fibre morphology and failure peculiarities suggested that the cell wall defects determined fibre strength. Fibre strength distribution function was derived taking into account the presence of a random number of defects with stochastic strength in a fibre. The reason of disagreement between experimentally determined elementary flax fibre strength and the Weibull strength distribution was tentatively thought related to particular features of defect distribution in the fibres. The obtained theoretical strength distribution function was shown to closely approximate the test data of flax fibres of different origins. A thorough experimental confirmation of the correlation between the distribution of defects in flax fibres and fibre strength distribution will be the subject of further research.

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